Plenary Talk 1

LOW-DIMENSIONAL PARADIGMS FOR HIGH-DIMENSIONAL HETERO-CHAOS

James Yorke
University of Maryland, USA

The dynamics on a chaotic attractor can be quite heterogeneous, being much more unstable in some regions than others. Some regions of a chaotic attractor can be expanding in more dimensions than other regions. Imagine a situation where two such regions and each contains trajectories that stay in the region for all time - while typical trajectories wander throughout the attractor. If furthermore arbitrarily close to each point of the attractor there are points on periodic orbits that have different unstable dimensions, then we say such an attractor is "hetero-chaotic" (i.e. it has heterogeneous chaos). This is hard to picture but we believe that most physical systems possessing a high-dimensional attractor are of this type. We have created simplified models with that behavior to give insight to real high-dimensional phenomena.

(This is joint work with Miguel Sanjuan, Yoshi Saiki, and Hiroki Takahashi)

Plenary Talk 2

DESCRIPTIVE THEORY OF DETERMINISTIC CHAOS

O.M.Sharkovsky
Institute of Mathematics, NAS of Ukraine

Descriptive theory of sets is a classical section of mathematics, which arose at the beginning of the last century. The talk proposes the basis of the **descriptive theory of chaos**.

We consider dynamical systems on a compact X, generated by a continuous map $f : X \to X$, mainly in the case of when X is an interval $I \subset R$.

The asymptotic behavior of every trajectory is usually determined through the socalled ω-limit set, or, more simply, **the attractor** of this trajectory. The set of all trajectories attracted by the same attractor is called **the basin of this attractor**.

Dynamical system if its topological entropy is positive:

1) has a lot of different attractors of trajectories, namely, the continuum of attractors;

2) basins of most attractors have a very complex structure, namely, they are sets of the 3rd class in the terminology of the descriptive theory of sets;

3) basins of different attractors are very intertwined and they can not be separated from each other by open or closed sets, but only by sets of the 2nd class of complexity, and

4) in the space of all closed subsets of the state space (with the Hausdorff metric), the set of all attractors is an attractor net (network) whose cells are formed by Cantor sets whose points are themselves attractors.

Most of the results presented in the talk were obtained and translated into English back in the 60's of the last century.

Plenary Talk 3

HAUSDORFF DIMENSION OF SOME NON-CONFORMAL REPELLERS

Karoly Simon
Budapest University of Technology and Economics, Hungary

The new results in this talk are joint with Balázs Bárány.

The motivation of our research was to answer a fractal image compression related question asked by Michael Barnsley.

To solve this problem we combine techniques from one-dimensional dynamics and fractal geometry. As an application of our results, we can compute the Hausdorff dimension of the graphs of some fractal interpolation functions and generalized Takagi functions.

More precisely, we compute the Hausdorff dimension of function graphs which are repellers of some piecewise affine, and piecewise expanding maps but the dynamics cannot be described by any subshift of finite type. The difficulties caused by the non-Markovian behavior of these repellers are tackled by approximation by Markovian systems, using a notion of the pressure introduced by Franz Hofbauer. Unfortunately, the computation of the Hausdorff dimension for piecewise affine repellers even if their dynamics can be described by a subshift of finite type, is still too difficult. So, we need to apply some tricks to convert this problem into the computation of the Hausdorff dimension of some self-affine attractors on the plane. After this we can use a very recent theorem of Balázs Bárány, Michael Hochman and Ariel Rappaport about the Hausdorff dimension of self-affiene sets on the plane. Using this, we obtain the Hausdorff dimension of the repellers in Michael Barnsley's question, mentioned above.

Plenary Talk 4 (Prof. K.R. Unni Memorial Lecture)

SPARSE VAR AND NETWORK REPRESENTATIONS OF HIGH DIMENSIONAL TIME SERIES DATA

Govindan Rangarajan
Indian Institute of Science, Bangalore, India

Obtaining a sparse representation of high dimensional data is important since it provides an efficient representation of the data and facilitates its further analysis. Conventional Vector Autoregressive (VAR) modeling methods applied to such data result in non-sparse solutions with a large number of spurious coefficients. We propose two sparse VAR modelling methods that work well for high dimensional time series data, even when the number of time points is relatively low, by incorporating only statistically significant coefficients. In numerical experiments, our methods show consistently higher accuracy compared to other contemporary methods in recovering the true sparse model. The relative absence of spurious coefficients in our models permits more accurate, stable and efficient evaluation of derived quantities such as power spectrum, coherence and Granger causality. Using our models, sparse functional connectivity networks can be computed, in a reasonable time, from data comprising tens of thousands of channels/voxels. This far exceeds the capabilities of existing methods and enables simultaneous analysis of both local and global functional connectivity patterns and community structures in such large networks. When computed for fMRI data, these network and community structures are consistent over independent recording sessions and they show good spatial correspondence with known functional and anatomical regions of the brain. Our methods, when used to analyze ADHD fMRI data, provide new ways of differentiating between ADHD and typically developing children using global and node-level network measures.

Plenary Talk 5

MATCHING FOR DISCONTINUOUS INTERVAL MAPS

Henk Bruin
University of Vienna, Austria

Entropy and properties of invariant densities are two ways of quantifying the amount of chaos that a dynamical system can exhibit. If this dynamical system is an iterated map on the interval, these things can be computed with some amount of precision, if the system possesses a Markov (i.e., invariant)
partition. Without the existence of such partition, these objects behaves quite irregularly as function of the parameters of the system. It is therefore curious to observe for certain families of interval maps, that, despite the lack of a Markov partition, the invariant density is piecewise smooth, and entropy piecewise monotone. The principle behind this is that certain important orbits will synchronize (called matching) for large parameter sets (matching set).

In this talk, I will discuss this phenomenon for a family of piecewise linear interval maps. Especially the non-matching parameter set becomes very interesting, and I will present results on this, which are joint work with Carlo Carminati, Alessandro Profetti, Stefano Marmi (Pisa).

Plenary Talk 6 (Prof. T.Thrivikraman Endowment Lecture)

INTERVAL MAPS WITH UNIQUE SELF-CONJUGACY

V.Kannan
University of Hyderabad, India

We ask a purely algebraic question: Which interval maps admit a unique self-conjugacy? Self-conjugacy of (R,f) means an order-isomorphism that commutes with f. The answer to this question turns out to be in terms of dynamical properties. We work among piecewise monotonic maps (say, for example, polynomials) whose every power has only finitely many fixed points. It will follow from the main theorem that every such chaotic map on the interval admints no self-conjugacy other than identity. It is a corollary that there is a unique order-conjugacy from the tent map to the logistic map.

Plenary Talk 7

A COMPUTER FRIENDLY ALTERNATIVE TO MORSE-NOVIKOV THEORY FOR TOPOLOGICAL CLOSED ONE FORM

Dan Burghelea
Ohio State University, USA

The concept of topological closed one form on a compact nice space (ANR) generalizes the concept of smooth closed one form on a smooth manifold and of one cocycle on a finite simplicial complex.

To such form, under mild hypothesis of tameness one extends the concept of "closed , open closed-open and open-closed bar codes familiar in the Zig-Zag persistence, providing new computable invariants from which one can recover up to isomorphism the Morse-Novikov complex in classical Morse-Novikov theory and for which one can prove strong stability properties and refined Poincar'e duality properties. These bar codes are actually real numbers (nonnegative for closed bar codes negative for open bar codes and positive resp. negative for closed –open and open-closed bar codes. They correspond to the sign length of the bar codes in case of real-valued or angle-valued maps They are in principle computable but reasonable algorithms remain an computational challenge. The topology behind the theory is conceptually elementary but tedious.

Plenary Talk 8 (Prof. T.A. Saraswathy Amma Memorial Lecture)

COARSE GEOMETRY AND QUASI-ISOMETRY GROUPS

Parameswaran Sankaran
Institute of Mathematical Sciences, Chennai, India

I will recall the notion of a quasi-isometry between two metric spaces. One has an equivalencerelation on the class of all self-quasi-isometries of a metric space X. The equivalence classes form a group called the quasi-isometry group of X. It is an interesting problem to study this group for important classes of metric spaces which arise naturally in geometry and in group theory. We will see examples of spaces for which the quasi-isometry groups are well understood and some important space for which the group is still mysterious.

Plenary Talk 9

MANDELPINSKI NECKLACES: STRUCTURES IN THE PARAMETER PLANE FOR SINGULARLY PERTURBED RATIONAL MAPS

Robert L. Devaney
Boston University, USA

In this lecture we consider rational maps of the form $z^n + C/z^n$ where $n > 2$. When C is small, the Julia sets for these maps are Cantor sets of circles and the corresponding region in the C-plane (the parameter plane) is the McMullen domain. We shall show that the McMullen domain is surrounded by infinitely many simple closed curves called Mandelpinski necklaces. The k^th necklace contains exactly $(n-2)n^k + 1$ parameters that are the centers of baby Mandelbrot sets and the same number of parameters that are centers of Sierpinski holes, i.e., disks in the parameter plane where the corresponding Julia sets are Sierpinski curves (sets that are homeomorphic to the Sierpinski carpet fractal). We shall also briefly describe other interesting structures in the parameter plane.

Plenary Talk 10

REGULAR AND SEMI REGULAR MINIMAL FLOWS

Jospeh Auslander
University of Maryland, USA

Let (X,T) be a minimal flow and let x,y in X. An obvious necessary condition for there to be an automorphism f of (X,T) with f(x)=y is that (x,y) be an almost periodic point of the product flow. The flow (X,T) is said to be regular if this is always the case. Regular minimal flows are the minimal left ideals of the enveloping semigroup of a flow.

A further necessary condition for the existence of an automorphism is given in terms of the automorphism group of the universal minimal flow, namely that if (m,n) projects to (x,y) and g(m)=n, then g must be in the normalizer of the Ellis group of (X,T). When this always occurs (X,T) is said to be semi regular. Every minimal flow has a semi regular one in the same proximal class.

Plenary Talk 11

GLOBAL DYNAMICS

Saber Elaydi
Trinity University, USA

We will survey the recent advances in the study of global stability and global dynamics of monotone discrete dynamical systems. Non-monotone amiable maps will be our next focus. Applications to population biology will be presented.

Invited Talk 1

REFLECTIONS ON ENVELOPING SEMIGROUPS

Anima Nagar
Indian Institute of Technology, Delhi, India

A flow is a pair (X,T) of compact metric space X and a topological group T acting on X, and topological dynamics is the study of the dynamics resulting from the action of T on X. Robert Ellis had defined and studied the algebraic properties of a flow (X,T) via the "Enveloping Semigroup". The Enveloping Semigroup $E(X)$ is defined as the closure of the set of homeomorphisms as an action of T on X in X^X with the product topology. In this talk we reflect into the dynamical properties of Enveloping Semigroups of dynamical flows and exhibit connections between these properties to some dynamical behavior of the flow.

Invited Talk 2

TOPOLOGICAL FEATURE-DIRECTED VISUALIZATION

Vijay Natarajan
Indian Institute of Science, Bangalore, India

Scientific phenomena are often studied through collections of related scalar fields generated from different observations of the same phenomenon. Exploration of such data requires a robust distance measure to compare scalar fields for tasks such as identifying key events and establishing correspondence between features within a data set and across data sets. In this talk, I will first introduce the problem of symmetry detection in scientific data and its role in the design of feature-directed visualization methods. The goal is to identify regions of interest within the domain of a scalar field that remain invariant under transformations of both domain geometry and the scalar values. The problem generalises to similarity identification when applied to time-varying and multi-field data. I will present algorithms to detect symmetry and similarity and discuss applications to visualization, interactive exploration, and visual analysis of large and feature-rich scientific data. [http://vgl.csa.iisc.ac.in]

Invited Talk 3

ON THE IMAGE OF THE JONES' SET FUNCTION T

Javier Camargo
Univeridad Industrial de Santander, Colombia

We present the Jones' set function T, showing some properties and examples. Particularly, we are interested in the continuity of the set function T. Furthermore, we show that it is interesting to study its image in some particular contexts; for instance when $T(2^X)$ is either connected or compact.

Invited Talk 4

TOPOLOGICAL ANALYSIS OF MULTIVARIATE DATA

Amit Chattopadhyay
International Institute of Information Technology, Banglaore, India

Topological Data Analysis (TDA) is a newly emerging branch of data science - where the goal is to understand the global shape of large, high-dimensional data by computing the topological features, namely, the number of connected components, tunnels, voids or higher-dimensional holes of the underlying shapes evolved in the data. Multivariate (or, multifield) data, that involves multiple scalar fields, occurs in many scientific experiments and simulations. For scalar data topological analysis has proven to be useful for extracting important features using tools like contour trees, Reeb graphs, Morse-Smale complexes and persistence diagram. However, multifield data is complex and extracting underlying topological and geometrical features needs progress both theoretically and computationally. This talk will give an overview of the recent developments in the multifield topological analysis.

Invited Talk 5

GENERAL POSITION THEOREMS IN THE STUDY OF NON-WSP SELF-SIMILAR SETS

Andrei Tetenov
Novosibirsk state university and Gorno-Altaisk state university, Russia

We prove and discuss General Position Theorem, which allows to construct families of self-similar sets with prescribed behavior of their critical sets. It gives exact overlap for double fixed points for twofold Cantor sets, allows to obtain one-point intersections for the pieces of self-similar arc in R^3 which does not satisfy OSC. This method is applied to prove the existence of systems in R with unique one point intersection, not satisfying WSP and to study the geometry of the set of critical parameters for parametrized families of self-similar sets. (This is a joint work with Kirill Kamalutdinov).

Invited Talk 6

SOME NEW RESULTS IN 2D PERSISTENT HOMOLOGY

Patrizio Frosini
University of Bologna, Italy

In this talk we will describe a novel approach to 2D sublevel set persistent homology. This approach is based on the concepts of extended Pareto grid, coherent transport of matchings between persistence diagrams associated with positive slope lines, persistent monodromy group, and coherent matching distance between 2D persistence diagrams. Some new results concerning this theoretical model, including a maximum principle, will be also illustrated. Finally, we will show how these mathematical tools can be used to cast new light on persistent homology for bifiltrations.

Invited Talk 7

THE AUSLANDER-YORKE DICHOTOMY: EQUICONTINUITY, SENSITIVITY AND TRANSITIVITY

Chris Good
University of Birmingham, UK

Let $f : X \to X$ be a continuous function on the compact metric space X. The AuslanderYorke Dichotomy states that if the system (X,f) is minimal, then it is either sensitive to initial conditions or equicontinuous.

It turns out that one can define notions of sensitivity and equicontinuity for continuous functions on compact Hausdorff spaces which are equivalent to the standard definitions in metric spaces. In this paper we take a look at the relationship between these topological notions of sensitivity, equicontinuity and transitivity. In doing so we uncover an Auslander-Yorke type dichotomy for transitive spaces.(This is a joint work with Joel mitchell).

Invited Talk 8

NEW CLASS OF PARACOMPACT SPACES IN TOPOLOGICAL SPACES

P. G. Patil
Karnatak University, Dharward, India

The purpose of this paper is to introduce and study the new class of paracompact spaces called $g^*\omega\alpha$-paracompact spaces as a generalization of paracompact spaces in topological spaces. We characterize $g^*\omega\alpha$-paracompact spaces and study their basic properties. (This is a joint work with S. Mirajakar).

Invited Talk 9

THE TESTING GROUND OF WEIGHTED SHIFT OPERATORS FOR HYPERCYCLICITY

Kit Chan
Bowling Green State University, USA

We explore the hypercyclicity of unilateral weighted backward shifts and bilateral weighted shifts on p, where $1 \leq p \leq \infty$, with the weak or weak-star topologies. Then we turn our attention to see how a nonzero limit point of an orbit of such an operator determines the hypercyclicity of the operator. Lastly, we explore a recent result that a unilateral weighted backward shift can be factored as the product of two hypercyclic shifts.

Invited Talk 10

MULTIVALUED INVERSE LIMITS

Wlodzimierz J Charatonik
Missouri University of Science & Technology, USA

We will discuss several topological properties that are preserved under multivalued inverse limits under specific assumptions on the bonding functions. Unlike for single valued case the inverse limit of continua does not have to be a continuum, so we discuss theorems that guarantee connectedness of inverse limits. Special attention is given to function where images or preimages of points are connected. For such functions we have theorems about connectedness, shape, local connectedness, hereditary unicouherence, decomposabitity, indecomposability, etc.

Invited Talk 11

WEAKLY CONTRACTIVE ITERATED FUNCTION SYSTEMS AND BEYOND

Krzysztof Lesniak
Nicolas Copernicus University in Torun, Poland

We give a systematic account of iterated function systems (IFS) of weak contractions of different types: Browder, Rakotch, Edelstein, topological. While many basic questions concerning these systems have already been addressed in the literature, there is still the potential for further investigation. We show that the existence of attractors and asymptotically stable invariant measures, and the validity of the random iteration algorithm a.k.a. “chaos game", can be obtained rather easily for weakly contractive systems. We show that the class of attractors of weakly contractive IFSs is essentially wider than the class of classical IFSs' fractals (in particular due to Dumitru, and Banakh & Nowak). On the other hand, we show (following results of D'Aniello & Steele and others) that in reasonable spaces, a typical compact set is not an attractor of any weakly contractive IFS. We explore the possibilities and restrictions to break the contractivity barrier by employing several tools from fixed point theory: geometry of balls (Lifshits constant), convolved contractivity (eventual contractions, average contractions), remetrization technique (results of Kameyama, Mihail & Miculescu, Banakh and others), ordered sets (Knaster-Tarski principle, Zorn lemma, results of Hayashi, Edalat and others), and measures of noncompactness (results of Ok and others). From these considerations it follows that while the existence of invariant sets and invariant measures can be assured rather easily for general iterated function systems under mild conditions, to establish the existence of attractors and unique invariant measures is a substantially more difficult problem. This explains the central role of contractive systems in the theory of iterated function systems.

Invited Talk 12

BOREL COMPLEXITY OF NORMAL NUMBERS VIA GENERIC POINTS IN SUBSHIFTS WITH SPECIFICATION

Dominik Kwietniak
Jagiellonian University in Krakow, Poland

We study the Borel complexity of sets of normal numbers in several numeration systems. Taking a dynamical point of view, we offer a unified treatment for continued fraction expansions and base b expansions, and their various generalisations: generalised Lüroth series expansions and β-expansions. In fact, we consider subshifts over a countable alphabet generated by all possible expansions of numbers in [0,1). Then normal numbers correspond to generic points of shift-invariant measures. It turns out that for these subshifts the set of generic points for a shift-invariant probability measure is precisely at the third level of the Borel hierarchy (it is a π^0_3-complete set, meaning that it is a countable intersection of F_σ sets, but it is not possible to write it as a countable union of G_δ-sets). We also solve Sharkovsky–Sivak problem on Borel complexity of the basin of statistical attraction. The crucial dynamical feature we need is a feeble form of specification. All expansions named above generate subshifts with this property. Hence sets of normal numbers under consideration are $mathbf\pi^0_3$-complete. (The talk is based on a joint work with: Dylan Airey, Steve Jackson, and Bill Mance).

Invited talk 13

ZERO – DIMENSIONAL COVERS OF DYNAMICAL SYSTEMS

Hisao Kato
Institute of Mathematics, University of Tsukuba, Japan

A pair (X,f) is called a *dynamical system* if X is a compact metric space (= compactum) and $f : X \to X$ is a map on X. A dynamical system $(Z,\tilde{f})$ *covers* (X,f) *via* a map $p : Z \to X$ provided that p is an onto map and $p\tilde{f} = fp$.

Note that (X,f) is also called a *factor* of $(Z,\tilde{f})$ and conversely $(Z,\tilde{f})$ is called a *cover* (or an *extension*) of (X,f). We call the map $p : Z \to X$ a *factor mapping*.
If Z is zero-dimensional, then we say that the dynamical system $(Z,\tilde{f})$ is a *zerodimensional cover* of (X,f). Moreover, if the factor mapping is a finite-to-one map, then we say that the dynamical system $(Z,\tilde{f})$ is a *finite-to-one zero-dimensional cover* of (X,f).

The (symbolic) dynamical systems on Cantor sets have been studied by many mathematicians and also the strong relations between Markov partitions and symbolic dynamics have been studied, R. D. Anderson proved that for any dynamical system (X,f), there exists a zero-dimensional cover $(Z,\tilde{f})$ of (X,f), and moreover M. Boyle, D. Fiebig and U. Fiebig proved that any dynamical system (X,f) has a zero-dimensional cover $(Z,\tilde{f})$ such that the topological entropy $h(f)$ of f is equal to $h(\tilde{f})$, where the factor mappings are not necessarily finite-to-one. In topology, there is a classical theorem by Hurewicz that any compactum X is at most n-dimensional if and only if there is a zero-dimensional compactum Z with an onto map $p : Z \to X$ whose fibers have cardinality at most $n + 1$. In the theory of dynamical systems, we have the related general problem:

Problem 1.1. *What kinds of dynamical systems can be covered by zero-dimensional dynamical systems via finite-to-one maps?*

The motivation for this problem comes from (symbolic) dynamics on Cantor sets. To study dynamical properties of the original dynamics (X,f), the finiteness of the fibers of the factor mapping may be very important and so, in this article we focus on the finiteness of fibers of factor mappings. Related to Problem 1.1, first Kulesza proved the following significant theorem:
Theorem 1.2. (Kulesza) *For each homeomorphism f on an n-dimensional compactum X with zero-dimensional set $P(f)$ of periodic points, there is a zerodimensional cover $(Z,\tilde{f})$ of (X,f) via an at most $(n + 1)^n$-to-one map such that $\tilde{f} : Z \to Z$ is a homeomorphism.*

He also showed that Problem 1.1 needs the assumption $\dim P(f) \leq 0$. In fact, for the disk $X = [0,1]^2$ or some 1-dimensional continuum X, there is a dynamical system (X,f) such that $f : X \to X$ is a homeomorphism on X with $\dim P(f) = 1$ and (X,f) has no zero-dimensional cover via a finite-to-one map. Ikegami, Kato and Ueda improved the theorem of Kulesza as follows: The condition of at most $(n + 1)^n$-to-one map can be strengthened to the condition of at most 2^n-to-one map.

The aim of this article is to give a partial answer to Problem 1.1. In fact, we show that the above theorem is also true for a class of maps containing two-sided zero-dimensional maps. For the special case that (X,f) is a positively expansive dynamical system with $\dim X = n$, (X,f) can be covered by a subshift (Σ,σ) of the shift map $\sigma : \{1,2,...,k\}^{\infty} \rightarrow \{1,2,...,k\}^{\infty}$ via an at most 2^n-to-one map. Also, we study some dynamical zero-dimensional decomposition theorems of spaces related to such maps.

Invited Talk 14

MINIMALITY FOR ACTIONS OF ABELIAN (SEMI) GROUPS

Roman Hric
Matej Bel University, Slovakia

I will present selected problems and results from a joint work with L’. Snoha: *Dense orbits in discrete and continuous systems on topological spaces* (Preprint 2018), and a joint work with M. Dirba´k, P. Maliˇcky´, L’. Snoha and V. Spitalsky´: *Minimality for actions of abelian semigroups on compact spaces with a free interval*, Ergodic Theory & Dynamical Systems (First published online 2018).

In the first work, we focus on topological transitivity and minimality of maps, homeomorphisms and (semi)flows on topological spaces—in general, noncompact ones. A special emphasis is given on hereditariness of density of orbits from (semi)flows to time-t maps.

In the second work, we investigate minimality of actions of general abelian semigroups on compact spaces possesing a free interval, i.e. an open subset of the space homeomorphic to the open interval. We characterise such spaces admitting a minimal action of a given abelian semigroup and we also describe the topological structure of minimal sets intersecting a free interval.