ME202: ADVANCED MECHANICS OF SOLIDS
MODULE – I

Introduction to Stress Analysis & Displacement Field
ME010 306(CE) STRENGTH OF MATERIALS & STRUCTURAL ENGINEERING

Course Objectives:

1. To impart concepts of stress and strain analyses in a solid.

2. To study the methodologies in theory of elasticity at a basic level.

3. To acquaint with the solution of advanced bending problems.

4. To get familiar with energy methods for solving structural mechanics problems.
Module – I

1. Introduction to stress analysis in elastic solids
2. Stress at a point - Stress tensor
3. Stress components in rectangular and polar coordinate systems
4. Cauchy’s equations
5. Stress transformation.
6. Principal stresses and planes.
7. Hydrostatic and Deviatoric stress components, Octahedral shear stress
8. Equations of equilibrium.
STRUCTURAL FAILURES
TYPES OF STRESSES
TENSION TEST:

Brittle Material

Ductile Material

Presented to S4 ME students of RSET by Dr. Manoj G Tharian
DUCTILE FAILURE

- Strain Hardening
- Necking
- Ultimate Strength
- Yield Strength
- Rise
- Run

Young's Modulus = \frac{\text{Rise}}{\text{Run}} = \text{Slope}

Stress vs. Strain graph showing the different stages of ductile failure and the calculation of Young's Modulus.
CUP AND CONE FORMATION:

Necking

Formation of Microvoids

Coalescence of microvoids to form cracks

DUCTILE FRACTURE

Crack Propogation by Shear deformation

Fracture
NATURE OF STRESS ON AN INCLINED PLANE:

On Plane \( n \) normal to the Applied Load:

\[ \sigma = \frac{P}{A} \quad \tau = 0 \]

On Plane \( n' \) inclined at an angle \( \theta \) to the applied load:

\[ \sigma = \frac{P_n}{A'} = \frac{PCos^2\theta}{A} \]
\[ \tau = \frac{P_s}{A'} = \frac{P.Sin\theta.Cos\theta}{A} \]
TRANSFORMATION EQU. IN PLANE STRESS:

\[
\sigma_{x1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos2\theta + \tau_{xy}\sin2\theta
\]

\[
\tau_{x1y1} = -\frac{\sigma_x - \sigma_y}{2} \sin2\theta + \tau_{xy}\cos2\theta
\]

\[
\sigma_{y1} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} \cos2\theta - \tau_{xy}\sin2\theta
\]
PRINCIPAL PLANE IN PLANE STRESS:

\[ Tan2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} \]

\[ \sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \]
The plane of max. shear in plane stress (2D) is given by:

$$\tan 2\theta_s = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}}$$

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$
Find the normal stress & shear stress on a 22.5° plane and also the principal stresses, principal planes, max shear stress and planes of max. shear stress for the following states of stress.

1. \( \sigma_x = -60 \text{ MPa} \quad \sigma_y = 0 \text{ MPa} \)  
   \( \tau_{xy} = 90 \text{ MPa} \)

2. \( \sigma_x = 45 \text{ MPa} \quad \sigma_y = 27 \text{ MPa} \)  
   \( \tau_{xy} = 18 \text{ MPa} \)
\[ \sigma_X = \tau_{XY} = 0 \]

\[ \tan 2\theta_P = \frac{2\tau_{xy}}{\sigma_X - \sigma_Y} \]

\[ \tan 2\theta_P = 0 \]

\[ 2\theta_P = 0, 180 \]

\[ \theta_P = 0, 90 \]

\[ \tan 2\theta_S = -\frac{\sigma_X - \sigma_Y}{2\tau_{xy}} \]

\[ \tan 2\theta_S = \infty \]

\[ 2\theta_S = 90, 270 \quad \theta_S = 45, 135 \]
3D STATE OF STRESS

Presented to S4 ME students of RSET by Dr. Manoj G Tharian

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Stress at a point is denoted by the stress tensor as given below:

\[
\begin{bmatrix}
\sigma_{xx} & \tau_{xy} & \tau_{xz} \\
\tau_{yx} & \sigma_{yy} & \tau_{yz} \\
\tau_{zx} & \tau_{zy} & \sigma_{zz}
\end{bmatrix}
\]

Or

\[
\begin{bmatrix}
\tau_{yx} & \sigma_{xy} & \tau_{yz} \\
\tau_{zx} & \tau_{zy} & \sigma_{zz}
\end{bmatrix}
\]

Or

\[
\begin{bmatrix}
\sigma_{xx} & \tau_{xy} & \tau_{xz} \\
\tau_{xy} & \sigma_{yy} & \tau_{yz} \\
\tau_{xz} & \tau_{yz} & \sigma_{zz}
\end{bmatrix}
\]
Consider a 3 D state of stress.

ABC is an arbitrary plane whose normal is n.

Direction Cosines of n are $n_x$, $n_y$ and $n_z$.

Plane ABC is at a distance of $h$ from Q

ABCD forms a tetrahedron.
The tetrahedron is isolated and a free body diagram is shown in Fig.
CAUCHY’S STRESS FORMULA

\( T^n \) - Resultant stress vector on the plane.

\( T^n_x \) - Component along x axis

\( T^n_y \) - Component along y axis

\( T^n_z \) - Component along z axis

A - Area of Plane ABC

Area of plane AQC = \( A n_x \)

Area of plane AQB = \( A n_y \)

Area of plane BQC = \( A n_z \)

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CAUCHY’S STRESS FORMULA

B_x, B_y, B_z – Body forces along x, y and z directions.

Volume of the tetrahedron = 1/3 Ah

Considering the equilibrium along the x, y and z axis we get,

\[ T^n_x A = \sigma_x A n_x + \tau_{xy} A n_y + \tau_{xz} A n_z - B_x \frac{1}{3} Ah \]

Cancelling all A’s and taking limit h→ 0, gives

\[ T^n_x = \sigma_x n_x + \tau_{xy} n_y + \tau_{xz} n_z \]

Similarly considering equilibrium along y and z axis gives

\[ T^n_y = \tau_{xy} n_x + \sigma_y n_y + \tau_{yz} n_z \]

\[ T^n_z = \tau_{xz} n_x + \tau_{yz} n_y + \sigma_z n_z \]
The above three equations are known as Cauchy’s Stress equations.

Cauchy’s stress equation can be written in the matrix form as

\[
\begin{bmatrix}
T_x^n \\
T_y^n \\
T_z^n
\end{bmatrix}
= 
\begin{bmatrix}
\sigma_x & \tau_{xy} & \tau_{xz} \\
\tau_{xy} & \sigma_y & \tau_{yz} \\
\tau_{xz} & \tau_{yz} & \sigma_z
\end{bmatrix}
\begin{bmatrix}
n_x \\
n_y \\
n_z
\end{bmatrix}
\]
The resultant stress vector on plane \( n \) is

\[
|T_n|^2 = T_{x_n}^2 + T_{y_n}^2 + T_{z_n}^2
\]

The normal stress and shear stress on plane \( n \) can be obtained using the following equations

\[
\sigma_n = n_x T_x^n + n_y T_y^n + n_z T_z^n
\]

\[
|T_n|^2 = \sigma_n^2 + \tau_n^2
\]
At a point Q in a body

\[ \sigma_x = 10000 \text{ N/cm}^2; \quad \sigma_y = -5000 \text{ N/cm}^2; \quad \sigma_z = -5000 \text{ N/cm}^2 \quad \tau_{xy} = \tau_{xz} = \tau_{xz} = 10000 \text{N/cm}^2 \]

Determine the normal and shear stress on a plane that is equally inclined to all three axes.
CAUCHY’S STRESS FORMULA - PROBLEM

\[ T_x^n = 17320.5 \text{ N/cm}^2 \]

\[ T_y^n = 8660.25 \text{ N/cm}^2 \]

\[ T_x^n = 8660.25 \text{ N/cm}^2 \]

\[ \sigma_n = 20000 \text{ N/cm}^2 \]

\[ |T^n|^2 = 450 \times 10^6 \]

\[ \tau^n = 7071 \text{ N/cm}^2 \]
PRINCIPAL STRESSES IN 3D STATE OF STRESS

A Plane where there is no shear stress is called a Principal Plane.
PRINCIPAL STRESSES IN 3D STATE OF STRESS

Let $n_x$, $n_y$ and $n_z$ be the Direction Cosines of the Principal Plane.

\[
\begin{align*}
T_x^n &= \sigma \cdot n_x \\
T_y^n &= \sigma \cdot n_y \\
T_z^n &= \sigma \cdot n_z
\end{align*}
\]

1

where, $\sigma$ is the Principal Stress.

Using Cauchy’s Equation,

\[
\begin{align*}
T_x^n &= \sigma_x n_x + \tau_{xy} n_y + \tau_{xz} n_z \\
T_y^n &= \tau_{xy} n_x + \sigma_y n_y + \tau_{yz} n_z \\
T_z^n &= \tau_{xz} n_x + \tau_{yz} n_y + \sigma_z n_z
\end{align*}
\]

2
PRINCIPAL STRESSES IN 3D STATE OF STRESS

Equating Eqs. 1 and 2 we get

\[ \sigma \cdot n_x = \sigma_x n_x + \tau_{xy} n_y + \tau_{xz} n_z \]

\[ \sigma \cdot n_y = \tau_{xy} n_x + \sigma_y n_y + \tau_{yz} n_z \]

\[ \sigma \cdot n_z = \tau_{xz} n_x + \tau_{yz} n_y + \sigma_z n_z \]

\[ (\sigma_x - \sigma) n_x + \tau_{xy} n_y + \tau_{xz} n_z = 0 \]

\[ \tau_{xy} n_x + (\sigma_y - \sigma) n_y + \tau_{yz} n_z = 0 \]

\[ \tau_{xz} n_x + \tau_{yz} n_y + (\sigma_z - \sigma) n_z = 0 \]
PRINCIPAL STRESSES IN 3D STATE OF STRESS

\[
\begin{vmatrix}
\sigma_x - \sigma & \tau_{xy} & \tau_{xz} \\
\tau_{xy} & \sigma_y - \sigma & \tau_{yz} \\
\tau_{xz} & \tau_{yz} & \sigma_z - \sigma
\end{vmatrix} = 0
\]

Expanding the above equation we get

\[
\sigma^3 - \left(\sigma_x + \sigma_y + \sigma_z\right)\sigma^2 +
\left(\sigma_x\sigma_y + \sigma_x\sigma_z + \sigma_y\sigma_z - \tau_{xy}^2 - \tau_{xz}^2 - \tau_{yz}^2\right)\sigma
- \left(\sigma_x\sigma_y\sigma_z + 2\tau_{xy}\tau_{xz}\tau_{yz} - \sigma_x\tau_{yz}^2 - \sigma_y\tau_{xz}^2 - \sigma_z\tau_{xy}^2\right) = 0
\]
Principal Stresses can be found out by solving the above cubical equation.

\[ \sigma^3 - I_1 \sigma^2 + I_2 \sigma - I_3 = 0 \]

\[ I_1 = \sigma_x + \sigma_y + \sigma_z \]

\[ I_2 = \begin{vmatrix} \sigma_x & \tau_{xy} \\ \tau_{xy} & \sigma_y \end{vmatrix} + \begin{vmatrix} \sigma_x & \tau_{xz} \\ \tau_{xz} & \sigma_z \end{vmatrix} + \begin{vmatrix} \sigma_y & \tau_{yz} \\ \tau_{yz} & \sigma_z \end{vmatrix} \]

\[ I_3 = \begin{vmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_y & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{vmatrix} \]
INVARINANTS OF STRESS

$I_1, I_2$ and $I_3$ are called Stress Invariants. They are called so because the values of $I_1, I_2, I_3$ do not change even if the reference coordinates are changed. In the cubical equ.

$$\sigma^3 - I_1 \sigma^2 + I_2 \sigma - I_3 = 0$$

$I_1$ - First Stress Invariant

$I_2$ - Second Stress Invariant

$I_3$ - Third Stress Invariant
IN männlich OF STRESS

Let x’, y’, z’ be another frame of reference at the same point. With respect to the frame of reference the stress state is given by,

\[
\begin{bmatrix}
\sigma_{x'} & \tau_{x'y'} & \tau_{x'z'} \\
\tau_{y'x'} & \sigma_{y'} & \tau_{y'z'} \\
\tau_{z'x'} & \tau_{z'y'} & \sigma_{z'}
\end{bmatrix}
\]
INVARINANTS OF STRESS

\[ I_1^1 = \sigma_x + \sigma_y + \sigma_z \]

\[ I_2^1 = \begin{vmatrix} \sigma_x & \tau_{x/y} \\ \tau_{x/y} & \sigma_y \end{vmatrix} + \begin{vmatrix} \sigma_x & \tau_{x/z} \\ \tau_{x/z} & \sigma_z \end{vmatrix} + \begin{vmatrix} \sigma_y & \tau_{y/z} \\ \tau_{y/z} & \sigma_z \end{vmatrix} \]

\[ I_3^1 = \begin{vmatrix} \sigma_x & \tau_{x/y} & \tau_{x/z} \\ \tau_{x/y} & \sigma_y & \tau_{y/z} \\ \tau_{x/z} & \tau_{y/z} & \sigma_z \end{vmatrix} \]
INVARIENTS OF STRESS

The principal stresses at a point depends only on the load exerted on the body and not on the co ordinates of reference describing the rectangular stress components hence,

\[ \sigma^3 - I_1 \sigma^2 + I_2 \sigma - I_3 = 0 \]
\[ \sigma^3 - I_1^1 \sigma^2 + I_2^1 \sigma - I_3^1 = 0 \]

must give same solutions for \( \sigma \). So the coefficients \( \sigma^2 \), \( \sigma \) and constant term in the two equus. must be equal. Thus

\[ I_1 = I_1^1; \quad I_2 = I_2^1; \quad I_3 = I_3^1 \]
INVARINANTS OF STRESS

Find the principal stresses and their planes for the following state of stress

\[
\begin{bmatrix}
1 & 2 & 1 \\
2 & 2 & -3 \\
1 & -3 & 4
\end{bmatrix}
\]
Theorem 1: If \( n \) and \( n' \) are two planes through the same point \( P \) with corresponding stress vectors \( T^n \) and \( T^{n'} \) Then the projection of \( T^n \) along \( n' \) is equal to the projection of \( T^{n'} \) along \( n \)

![Diagram showing planes and stress vectors](image)

Theorem 2: Principal planes are orthogonal.
STRESS TRANSFORMATION

\[
\begin{bmatrix}
\sigma_x & \tau_{xy} & \tau_{xz} \\
\tau_{xy} & \sigma_y & \tau_{yz} \\
\tau_{xz} & \tau_{yz} & \sigma_z
\end{bmatrix} \rightarrow
\begin{bmatrix}
\sigma_{x'} & \tau_{x'y'} & \tau_{x'z'} \\
\tau_{y'x'} & \sigma_{y'} & \tau_{y'z'} \\
\tau_{z'x'} & \tau_{z'y'} & \sigma_{z'}
\end{bmatrix}
\]
Direction Cosines of x' be $n_{xx'}$, $n_{yx'}$, $n_{zx'}$

Direction Cosines of y' be $n_{xy'}$, $n_{yy'}$, $n_{zy'}$

Direction Cosines of z' be $n_{xz'}$, $n_{yz'}$, $n_{zz'}$

$n_{xx'}$ - Cos of angle between x and x'

$n_{yx'}$ - Cos of angle between y and x'

$n_{zx'}$ - Cos of angle between z and x'
While taking the sign of angle in xy plane anticlockwise direction is taken as positive while looking to the xy plane from the +ve z axis.

According to Caushy’s equation

\[
\begin{align*}
T_{x}^{x} &= \sigma_{x}n_{xx'} + \tau_{xy}n_{yx'} + \tau_{xz}n_{xz'}, \\
T_{y}^{x} &= \tau_{xy}n_{xx'} + \sigma_{y}n_{yx'} + \tau_{yz}n_{xz'}, \\
T_{z}^{x} &= \tau_{xz}n_{xx'} + \tau_{yz}n_{yx'} + \sigma_{z}n_{xz'}
\end{align*}
\]

(1)
STRESS TRANSFORMATION

For getting the component of $T^x'$ along the $x'$ direction take the dot product of $T^x'$ and $x'$

For getting the component of $T^x'$ along the $y'$ direction take the dot product of $T^x'$ and $y'$

For getting the component of $T^x'$ along the $z'$ direction take the dot product of $T^x'$ and $z'$

\[
\begin{align*}
\sigma_{x'} &= T^x_{x'} n_{xx'} + T^x_{y'} n_{yx'} + T^x_{z'} n_{zx'}, \\
\tau_{x'y'} &= T^x_{x} n_{xy'} + T^x_{y} n_{yy'} + T^x_{z} n_{zy'}, \\
\tau_{x'z'} &= T^x_{x} n_{xz'} + T^x_{y} n_{yz'} + T^x_{z} n_{zz'}
\end{align*}
\]
STRESS TRANSFORMATION

Substituting for $T_{x'}$, $T_{y'}$, $T_{z'}$ from equ 1 in equ 2 will give,

$$\sigma_{x'/x'} = \sigma_{xx}n_{xx'}^2 + \sigma_{yy}n_{yx'}^2 +$$

$$\sigma_{zz}n_{zx'}^2 + 2\tau_{xy}n_{xx'}n_{yx'} + 2\tau_{xz}n_{xx'}n_{zx'} + 2\tau_{yz}n_{yx'}n_{zx'}$$

$$\tau_{x'y'} = \sigma_{xx}n_{xx'}n_{xy'} + \sigma_{yy}n_{yx'}n_{yy'} + \sigma_{zz}n_{zx'}n_{zy'} +$$

$$\tau_{xy}(n_{xx'}n_{yy'} + n_{xy'}n_{yx'}) + \tau_{yz}(n_{yx'}n_{zy'} + n_{zx'}n_{yy'}) +$$

$$\tau_{xz}(n_{xx'}n_{zy'} + n_{zx'}n_{xy'})$$

$$\tau_{x'z'} = \sigma_{xx}n_{xx'}n_{xz'} + \sigma_{yy}n_{yx'}n_{yz'} + \sigma_{zz}n_{zx'}n_{zz'} +$$

$$\tau_{xy}(n_{xx'}n_{yz'} + n_{yx'}n_{xz'}) + \tau_{yz}(n_{yx'}n_{zz'} + n_{zx'}n_{yz'}) +$$

$$\tau_{xz}(n_{xx'}n_{zz'} + n_{zx'}n_{xz'})$$
STRESS TRANSFORMATION

The above set of three equations can be written in the matrix form as

$$\begin{align*}
\begin{pmatrix}
\sigma_{x'x'} \\
\tau_{x'y'} \\
\tau_{x'z'}
\end{pmatrix}
&= 
\begin{pmatrix}
 n_{xx'} & n_{yx'} & n_{zx'} \\
 n_{xy'} & n_{yy'} & n_{zy'} \\
 n_{xz'} & n_{yz'} & n_{zz'}
\end{pmatrix}
\begin{pmatrix}
\sigma_x & \tau_{xy} & \tau_{xz} \\
\tau_{xy} & \sigma_y & \tau_{yz} \\
\tau_{xz} & \tau_{yz} & \sigma_z
\end{pmatrix}
\begin{pmatrix}
 n_{xx'} \\
n_{yx'} \\
n_{zx'}
\end{pmatrix}
\end{align*}$$

$$\{\sigma\}_{x'} = [\alpha]^T [\sigma]_{xyz} \{n\}_{x'} \quad \text{----------------} \quad (3)$$

- Stress components on $x'$ plane
- Stress components on $xyz$ plane
- Direction Cosines of $x'$
STRESS TRANSFORMATION

\[
\begin{bmatrix}
 n_{xx} & n_{xy} & n_{xz} \\
 n_{yx} & n_{yy} & n_{yz} \\
 n_{zx} & n_{zy} & n_{zz}
\end{bmatrix}
\]

\[
\{\sigma\}_{y'} = [\alpha]^T [\sigma]_{xyz} \{n\}_{y'}
\] \hspace{1cm} (4)

The Stresses on the z’ plane is obtained as,

\[
\{\sigma\}_{z'} = [\alpha]^T [\sigma]_{xyz} \{n\}_{z'}
\] \hspace{1cm} (5)

The Stresses on the y’ plane is obtained as,
Combining equs. 3, 4 & 5

The Stress transformation equation is obtained:

\[
\{\sigma\}_{x'y'z'} = [\alpha]^T [\sigma]_{xyz} [\alpha]
\]
STRESS TRANSFORMATION

A state of stress at a point with respect to xyz is given by,

\[
\sigma = \begin{bmatrix}
10 & 6 & -8 \\
6 & 20 & -4 \\
-8 & -4 & 10
\end{bmatrix} \text{ MPa}
\]

Find the state of stress for new set of axis rotated about x axis to an angle 45°.
STRESS TRANSFORMATION

\[
\begin{bmatrix}
\sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\
\sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\
\sigma_{zx} & \sigma_{zy} & \sigma_{zz}
\end{bmatrix}
= \begin{bmatrix}
10 & 6 & -8 \\
6 & 20 & -4 \\
-8 & -4 & 10
\end{bmatrix} \text{ MPa}
\]

(x')   (y')   (z')

\[
\begin{bmatrix}
x' \\
y' \\
z'
\end{bmatrix}
= \begin{bmatrix}
x \\
y \\
z
\end{bmatrix}
\]
STRESS TRANSFORMATION

\[
[\sigma]_{xyz} = \begin{bmatrix}
10 & 6 & -8 \\
6 & 20 & -4 \\
-8 & -4 & 10
\end{bmatrix} \text{ MPa}
\]

\[
[\alpha] = \begin{bmatrix}
\cos(0) & \cos(90) & \cos(-90) \\
\cos(-90) & \cos(45) & \cos(135) \\
\cos(90) & \cos(-45) & \cos(45)
\end{bmatrix}
\]

\[
(x') \quad (y') \quad (z')
\]

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STRESS TRANSFORMATION

\[
\begin{bmatrix}
\sigma_{xyz}
\end{bmatrix} = \begin{bmatrix}
10 & 6 & -8 \\
6 & 20 & -4 \\
-8 & -4 & 10
\end{bmatrix} \text{ MPa}
\]

\[
\begin{bmatrix}
(x') \\
y' \\
(z')
\end{bmatrix} = \begin{bmatrix}
1 & 0 & 0 \\
0 & 0.7071 & -0.7071 \\
0 & 0.7071 & 0.7071
\end{bmatrix} \begin{bmatrix}
x \\
y \\
z
\end{bmatrix}
\]
DIFFERENTIAL EQUATION OF EQUILIBRIUM

Let the body force components per unit volume in the x, y and z direction be \( B_x \), \( B_y \) and \( B_z \).

For equilibrium along x direction,

\[
\left( \sigma_x + \frac{\partial \sigma_x}{\partial x} \Delta x \right) \Delta y \Delta z - \sigma_x \Delta y \Delta z +
\]

\[
\left( \tau_{yx} + \frac{\partial \tau_{yx}}{\partial y} \Delta y \right) \Delta x \Delta z - \tau_{yx} \Delta x \Delta z +
\]

\[
\left( \tau_{zx} + \frac{\partial \tau_{zx}}{\partial z} \Delta z \right) \Delta x \Delta y - \tau_{zx} \Delta x \Delta y + B_x \Delta x \Delta y \Delta z = 0
\]
Differential Equation of Equilibrium

Dividing by $\Delta x\Delta y\Delta z$ and taking the limits $\Delta x\Delta y\Delta z$ tends to zero

\[
\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} + B_x = 0
\]
DIFFERENTIAL EQUATION OF EQUILIBRIUM

\[
\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} + B_x = 0
\]

\[
\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{zy}}{\partial z} + B_y = 0
\]

\[
\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_z}{\partial z} + B_z = 0
\]

Equilibrium Equation is also called differential equation of motion for a deformable body.
A cross section of wall of dam is showed in fig. The pressure of water on face OB is also shown in fig. The stress at any point $xy$ are given below $\gamma$ – Specific weight of water, $\rho$ – specific weight of dam material.

$$\sigma_x = -\gamma y$$

$$\sigma_y = \left(\frac{\rho}{\tan\beta} - \frac{2\gamma}{\tan^3\beta}\right)x + \left(\frac{\gamma}{\tan^2\beta} - \rho\right)y$$

$$\tau_{xy} = -\frac{\gamma}{\tan^2\beta}x$$
HYDROSTATIC AND DEVIATORIC STATE OF STRESS

\[
[\sigma] = \begin{bmatrix}
\sigma_{xx} & \tau_{xy} & \tau_{xz} \\
\tau_{yx} & \sigma_{yy} & \tau_{yz} \\
\tau_{zx} & \tau_{zy} & \sigma_{zz}
\end{bmatrix}
\]

\[
= \begin{bmatrix}
P & 0 & 0 \\
0 & P & 0 \\
0 & 0 & P
\end{bmatrix} + \begin{bmatrix}
\sigma_{xx} - P & \tau_{xy} & \tau_{xz} \\
\tau_{yx} & \sigma_{yy} - P & \tau_{yz} \\
\tau_{zx} & \tau_{zy} & \sigma_{zz} - P
\end{bmatrix}
\]

Hydrostatic State. Deviatoric state.

Where, \( P = \frac{1}{3} \left[ \sigma_{xx} + \sigma_{yy} + \sigma_{zz} \right] = \frac{1}{3} I_1 \)
OCTAHEDRAL STRESSES

Consider the principal directions as the coordinate axes. The plane whose normal vector forms equal angles with the coordinate system is called octahedral plane. There are eight such planes forming an octahedron.
OCTAHEDRAL STRESSES

\[
\begin{bmatrix}
\sigma_1 & 0 & 0 \\
0 & \sigma_2 & 0 \\
0 & 0 & \sigma_3
\end{bmatrix}
\]

\[n_1 = n_2 = n_3 = \frac{1}{\sqrt{3}}\]

\[\sigma_{oct} = \frac{1}{3} \left[ \sigma_1 + \sigma_2 + \sigma_3 \right] = \frac{1}{3} I_1\]

\[\tau_{oct} = \frac{1}{3} \sqrt{[ (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 ]} = \frac{1}{3} \sqrt{2I_1^2 - 6I_2} = \sqrt{\frac{2}{3} I'_2}\]

Where \( I' \) is the second invariant of deviatoric stress tensor
Calculate the octahedral stresses for the following stress tensor:

\[ \sigma_{ij} = \begin{bmatrix} 4 & -2 & -1 \\ -2 & 3 & 4 \\ -1 & 4 & -2 \end{bmatrix} \]

Calculate the stress deviator tensor and its invariants for the following stress tensor:

\[ \sigma_{ij} = \begin{bmatrix} 2 & -3 & 4 \\ -3 & -5 & 1 \\ 4 & 1 & 6 \end{bmatrix} \]
Displacement Field

The displacement undergone by any point on a body can be expressed as a function of original coordinates. The displacement field $U$ is expressed as

$$U = u_i + v_j + w_k$$

This function is known as displacement field vector where,

$$u = f_1(x,y,z)$$

$$v = f_2(x,y,z)$$

$$w = f_3(x,y,z)$$
Strain Component along x direction,

\[ \varepsilon_{xx} = \frac{\partial u}{\partial x} + \frac{1}{2} \left[ \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial v}{\partial x} \right)^2 + \left( \frac{\partial w}{\partial x} \right)^2 \right] \]

Strain Component along y direction,

\[ \varepsilon_{yy} = \frac{\partial v}{\partial y} + \frac{1}{2} \left[ \left( \frac{\partial u}{\partial y} \right)^2 + \left( \frac{\partial v}{\partial y} \right)^2 + \left( \frac{\partial w}{\partial y} \right)^2 \right] \]

Strain Component along z direction,

\[ \varepsilon_{zz} = \frac{\partial w}{\partial z} + \frac{1}{2} \left[ \left( \frac{\partial u}{\partial z} \right)^2 + \left( \frac{\partial v}{\partial z} \right)^2 + \left( \frac{\partial w}{\partial z} \right)^2 \right] \]
STRAIN COMPONENTS  (STRAIN – DISPLACEMENT RELATIONS)

Shear Strain in the $xy$ plane,

$$\gamma_{xy} = 2\varepsilon_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial u}{\partial x}\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\frac{\partial v}{\partial y} + \frac{\partial w}{\partial x}\frac{\partial w}{\partial y}$$

Shear Strain in the $yz$ plane,

$$\gamma_{yz} = 2\varepsilon_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} + \frac{\partial u}{\partial y}\frac{\partial u}{\partial z} + \frac{\partial v}{\partial y}\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\frac{\partial w}{\partial z}$$

Shear Strain in the $xz$ plane,

$$\gamma_{xz} = 2\varepsilon_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} + \frac{\partial u}{\partial x}\frac{\partial u}{\partial z} + \frac{\partial v}{\partial x}\frac{\partial v}{\partial z} + \frac{\partial w}{\partial x}\frac{\partial w}{\partial z}$$
STRAIN DISPLACEMENT RELATIONS – (LINEAR TERMS ONLY)

\[ \varepsilon_{xx} = \frac{\partial u}{\partial x} \]

\[ \varepsilon_{yy} = \frac{\partial v}{\partial y} \]

\[ \varepsilon_{zz} = \frac{\partial w}{\partial z} \]

\[ \gamma_{xy} = 2\varepsilon_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \]

\[ \gamma_{yz} = 2\varepsilon_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \]

\[ \gamma_{xz} = 2\varepsilon_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \]
STATE OF STRAIN AND STRAIN TENSOR AT A POINT

\[
\begin{bmatrix}
\varepsilon_{xx} & \gamma_{xy} & \gamma_{xz} \\
\gamma_{xy} & \varepsilon_{yy} & \gamma_{yz} \\
\gamma_{xz} & \gamma_{yz} & \varepsilon_{zz}
\end{bmatrix}
\quad
\begin{bmatrix}
\varepsilon_{xx} & 1/2 \gamma_{xy} & 1/2 \gamma_{xz} \\
1/2 \gamma_{xy} & \varepsilon_{yy} & 1/2 \gamma_{yz} \\
1/2 \gamma_{xz} & 1/2 \gamma_{yz} & \varepsilon_{zz}
\end{bmatrix}
\]

State of Strain at a point

Strain Tensor

Presented to S4 ME students of RSET by Dr. Manoj G Tharian
Presented to S4 ME students of RSET by Dr. Manoj G Tharian

24th January 2019

ANALOGY BETWEEN STRESS AND STRAIN TENSOR - 1

\[
\begin{bmatrix}
\sigma_{xx} & \tau_{xy} & \tau_{xz} \\
\tau_{yx} & \sigma_{yy} & \tau_{yz} \\
\tau_{zx} & \tau_{zy} & \sigma_{zz}
\end{bmatrix}
\]

\[
\begin{bmatrix}
\varepsilon_{xx} & \frac{1}{2} \gamma_{xy} & \frac{1}{2} \gamma_{xz} \\
\frac{1}{2} \gamma_{yx} & \varepsilon_{yy} & \frac{1}{2} \gamma_{yz} \\
\frac{1}{2} \gamma_{zx} & \frac{1}{2} \gamma_{zy} & \varepsilon_{zz}
\end{bmatrix}
\]

STRESS TENSOR

\[
\tau_{xy} = \tau_{yx} ; \tau_{xz} = \tau_{zx} ; \tau_{zx} = \tau_{zy}
\]

\[
\gamma_{xy} = \gamma_{yx} ; \gamma_{yz} = \gamma_{zy} ; \gamma_{xz} = \gamma_{zx}
\]

STRAIN TENSOR
ANALOGY BETWEEN STRESS AND STRAIN TENSOR - 2

\[ \varepsilon_{PQ} = \varepsilon_{xx}n_x^2 + \varepsilon_{yy}n_y^2 + \varepsilon_{zz}n_z^2 + 2 \times \left( \frac{1}{2} \gamma_{xy} n_x n_y \right) + 2 \times \left( \frac{1}{2} \gamma_{yz} n_y n_z \right) + 2 \times \left( \frac{1}{2} \gamma_{xz} n_x n_z \right) \]

This is analogous to the stress relation:

\[ \sigma_n = n_x T^n_x + n_y T^n_y + n_z T^n_z \]

\[ \sigma^n = \sigma_{xx} n_x^2 + \tau_{xy} n_x n_y + \tau_{xz} n_x n_z + \sigma_{yy} n_y^2 + \tau_{yx} n_y n_x \]

\[ + \tau_{yz} n_y n_z + \sigma_{zz} n_z^2 + \tau_{zx} n_z n_x + \tau_{zy} n_z n_y \]

\[ \sigma^n = \sigma_{xx} n_x^2 + \sigma_{yy} n_y^2 + \sigma_{zz} n_z^2 + 2\tau_{xy} n_x n_y + 2\tau_{yz} n_y n_z + 2\tau_{xz} n_x n_z \]
ANALOGY BETWEEN STRESS AND STRAIN TENSOR - 3

STRAIN TRANSFORMATION EQUATION

The above Transformation equations can be written in the Matrix Form as:

\[
[\varepsilon]_{x'y'z'} = [\alpha]^T [\varepsilon] [\alpha]
\]

This is analogous to

\[
[\sigma]_{x'y'z'} = [\alpha]^T [\sigma] [\alpha]
\]

\[
[\varepsilon]_{xyz} = \begin{bmatrix}
\varepsilon_{xx} & \frac{1}{2} \gamma_{xy} & \frac{1}{2} \gamma_{xz} \\
\frac{1}{2} \gamma_{xy} & \varepsilon_{yy} & \frac{1}{2} \gamma_{yz} \\
\frac{1}{2} \gamma_{xz} & \frac{1}{2} \gamma_{yz} & \varepsilon_{zz}
\end{bmatrix}
\]

\[
[\alpha] = \begin{bmatrix}
n_{xx'} & n_{xy'} & n_{xz'} \\
n_{yx'} & n_{yy'} & n_{yz'} \\
n_{zx'} & n_{zy'} & n_{zz'}
\end{bmatrix}
\]
### ANALOGY BETWEEN STRESS AND STRAIN TENSOR - 4

#### PRINCIPAL STRAIN

\[
\begin{vmatrix}
\varepsilon_{xx} - \varepsilon \\ \frac{1}{2} \gamma_{xy} \\ \frac{1}{2} \gamma_{xz}
\end{vmatrix}
\begin{vmatrix}
\frac{1}{2} \gamma_{xy} \\ \varepsilon_{yy} - \varepsilon \\ \frac{1}{2} \gamma_{yz}
\end{vmatrix}
\begin{vmatrix}
\frac{1}{2} \gamma_{xz} \\ \frac{1}{2} \gamma_{yz} \\ \varepsilon_{zz} - \varepsilon
\end{vmatrix} = 0
\]

\[
\varepsilon^3 - J_1 \varepsilon^2 + J_2 \varepsilon - J_3 = 0
\]

\[
J_1 = \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz}
\]

\[
J_2 =
\begin{vmatrix}
\varepsilon_{xx} \\ \frac{1}{2} \gamma_{xy} \\ \frac{1}{2} \gamma_{xz}
\end{vmatrix}
\begin{vmatrix}
\frac{1}{2} \gamma_{xy} \\ \varepsilon_{yy} \\ \frac{1}{2} \gamma_{yz}
\end{vmatrix}
\begin{vmatrix}
\frac{1}{2} \gamma_{xz} \\ \frac{1}{2} \gamma_{yz} \\ \varepsilon_{zz}
\end{vmatrix}
\]

\[
J_3 =
\begin{vmatrix}
\varepsilon_{xx} \\ \frac{1}{2} \gamma_{xy} \\ \frac{1}{2} \gamma_{xz}
\end{vmatrix}
\begin{vmatrix}
\frac{1}{2} \gamma_{xy} \\ \varepsilon_{yy} \\ \frac{1}{2} \gamma_{yz}
\end{vmatrix}
\begin{vmatrix}
\frac{1}{2} \gamma_{xz} \\ \frac{1}{2} \gamma_{yz} \\ \varepsilon_{zz}
\end{vmatrix}
\]

24th January 2019

**Presented to S4 ME students of RSET**

*by Dr. Manoj G Tharian*
PRINCIPAL STRAIN

\[(\varepsilon_{xx} - \varepsilon)n_x + \frac{1}{2}\gamma_{xy}n_y + \frac{1}{2}\gamma_{xz}n_z = 0\]

\[\frac{1}{2}\gamma_{yx}n_x + (\varepsilon_{yy} - \varepsilon)n_y + \frac{1}{2}\gamma_{yz}n_z = 0\]

\[\frac{1}{2}\gamma_{zx}n_x + \frac{1}{2}\gamma_{zy}n_y + (\varepsilon_{zz} - \varepsilon)n_z = 0\]

\[n_x^2 + n_y^2 + n_z^2 = 1\]
Stress invariants in terms of Principal Stresses:

\[ I_1 = \sigma_1 + \sigma_2 + \sigma_3 \]
\[ I_2 = \sigma_1 \sigma_2 + \sigma_2 \sigma_3 + \sigma_1 \sigma_3 \]
\[ I_3 = \sigma_1 \sigma_2 \sigma_3 \]

Strain invariants in terms of Principal Strains:

\[ J_1 = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 \]
\[ J_2 = \varepsilon_1 \varepsilon_2 + \varepsilon_2 \varepsilon_3 + \varepsilon_1 \varepsilon_3 \]
\[ J_3 = \varepsilon_1 \varepsilon_2 \varepsilon_3 \]
RIGID BODY ROTATIONS

Rotation about the x axis:

\[ \omega_x = \frac{1}{2} \left( \frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} \right) \]
RIGID BODY ROTATIONS

Rotation about the y axis:

$$\omega_y = \frac{1}{2} \left( \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} \right)$$
Rotation about the z axis:

\[ \omega_z = \frac{1}{2} \left( \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) \]
MAX SHEAR STRESS & STRAIN

Maximum Shear Stress:

\[ \tau_{\text{max}} = \frac{\sigma_{\text{max}} - \sigma_{\text{min}}}{2} \]

Maximum Shear Strain:

\[ \frac{1}{2} \gamma_{\text{max}} = \frac{\varepsilon_{\text{max}} - \varepsilon_{\text{min}}}{2} \]

Principal Strain for 2D state of strain

\[ \varepsilon_1 = \frac{\varepsilon_{xx} + \varepsilon_{yy}}{2} + \sqrt{\left( \frac{\varepsilon_{xx} - \varepsilon_{yy}}{2} \right)^2 + \left( \frac{1}{2} \gamma_{xy} \right)^2} \]

\[ \varepsilon_1 = \frac{\varepsilon_{xx} + \varepsilon_{yy}}{2} - \sqrt{\left( \frac{\varepsilon_{xx} - \varepsilon_{yy}}{2} \right)^2 + \left( \frac{1}{2} \gamma_{xy} \right)^2} \]

\[ \tan 2\theta = \frac{\gamma_{xy}}{\varepsilon_{xx} - \varepsilon_{yy}} \]
The state of strain at a point is given by

\[
\varepsilon_{ij} = \begin{bmatrix}
0.02 & -0.04 & 0 \\
-0.04 & 0.06 & -0.02 \\
0 & -0.02 & 0 \\
\end{bmatrix}
\]

In the direction PQ having cosines \( nx = 0.6 \); \( ny = 0 \) and \( nz = 0.8 \); Determine \( \varepsilon_{PQ} \)
The displacement field for a body is given below

\[ U = (x^2 + y)i + (3 + z)j + (x^2 + 2y)k \]

Determine the principal strains at (3,1,-2) and the direction of minimum strain. Use only linear terms.
COMPATIBILITY CONDITIONS
COMPATIBILITY CONDITIONS

\[ \varepsilon_{xx} = \frac{\partial u}{\partial x} \]

\[ \varepsilon_{yy} = \frac{\partial v}{\partial y} \]

\[ \varepsilon_{zz} = \frac{\partial w}{\partial z} \]

\[ \gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \]

\[ \gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \]

\[ \gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \]
COMPATIBILITY CONDITIONS

The three displacement components and six strain components are related by six strain displacement relations of Cauchy.

The determination of six strain components from three displacement components involves only differentiation.

However the reverse operation that is determination of three displacement components from six strain components is more complicated. Since it involves integrating six equations to obtain 3 functions.
COMPATIBILITY CONDITIONS

Therefore all strain components cannot be prescribed arbitrarily and there must exist a definite relation among the strain components. This relation among strain components is called **compatibility equations**
COMPATIBILITY CONDITIONS

\[ \frac{\partial^2 \varepsilon_{xx}}{\partial y^2} = \frac{\partial^2}{\partial x \partial y} \left( \frac{\partial u}{\partial y} \right) \]

\[ \frac{\partial^2 \varepsilon_{yy}}{\partial x^2} = \frac{\partial^2}{\partial x \partial y} \left( \frac{\partial v}{\partial x} \right) \rightarrow \frac{\partial^2 \varepsilon_{xx}}{\partial y^2} + \frac{\partial^2 \varepsilon_{yy}}{\partial x^2} = \frac{\partial^2}{\partial x \partial y} \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \]

\[ \frac{\partial^2 \varepsilon_{xx}}{\partial y^2} + \frac{\partial^2 \varepsilon_{yy}}{\partial x^2} = \frac{\partial^2 \gamma_{xy}}{\partial x \partial y} \]

\[ \frac{\partial^2 \varepsilon_{yy}}{\partial z^2} + \frac{\partial^2 \varepsilon_{zz}}{\partial y^2} = \frac{\partial^2 \gamma_{yz}}{\partial y \partial z} \]

\[ \frac{\partial^2 \varepsilon_{xx}}{\partial z^2} + \frac{\partial^2 \varepsilon_{zz}}{\partial x^2} = \frac{\partial^2 \gamma_{xz}}{\partial x \partial z} \]
COMPATIBILITY CONDITIONS

\[ \gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \quad \gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \quad \gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \]

\[ \frac{\partial \gamma_{xy}}{\partial z} = \frac{\partial^2 u}{\partial y \partial z} + \frac{\partial^2 v}{\partial x \partial z} \]

\[ \frac{\partial \gamma_{yz}}{\partial x} = \frac{\partial^2 v}{\partial x \partial z} + \frac{\partial^2 w}{\partial x \partial y} \]

\[ \frac{\partial \gamma_{xz}}{\partial y} = \frac{\partial^2 u}{\partial y \partial z} + \frac{\partial^2 w}{\partial x \partial y} \]
COMPATIBILITY CONDITIONS

\[ \frac{\partial \gamma_{yz}}{\partial x} + \frac{\partial \gamma_{xz}}{\partial y} - \frac{\partial \gamma_{xy}}{\partial z} = 2 \frac{\partial^2 w}{\partial x \partial y} \]

\[ \frac{\partial}{\partial z} \left( \frac{\partial \gamma_{yz}}{\partial x} + \frac{\partial \gamma_{xz}}{\partial y} - \frac{\partial \gamma_{xy}}{\partial z} \right) = 2 \frac{\partial^3 w}{\partial x \partial y \partial z} \]

\[ \frac{\partial}{\partial z} \left( \frac{\partial \gamma_{yz}}{\partial x} + \frac{\partial \gamma_{xz}}{\partial y} - \frac{\partial \gamma_{xy}}{\partial z} \right) = 2 \frac{\partial^2 \varepsilon_{zz}}{\partial x \partial y} \]

\[ \frac{\partial}{\partial y} \left( \frac{\partial \gamma_{xy}}{\partial z} + \frac{\partial \gamma_{yz}}{\partial x} - \frac{\partial \gamma_{xz}}{\partial y} \right) = 2 \frac{\partial^2 \varepsilon_{yy}}{\partial x \partial z} \]

\[ \frac{\partial}{\partial x} \left( \frac{\partial \gamma_{xy}}{\partial z} + \frac{\partial \gamma_{xz}}{\partial y} - \frac{\partial \gamma_{yz}}{\partial x} \right) = 2 \frac{\partial^2 \varepsilon_{xx}}{\partial y \partial z} \]
COMPATIBILITY CONDITIONS

\[ \frac{\partial^2 \varepsilon_{xx}}{\partial y^2} + \frac{\partial^2 \varepsilon_{yy}}{\partial x^2} = \frac{\partial^2 \gamma_{xy}}{\partial x \partial y} \]

\[ \frac{\partial^2 \varepsilon_{yy}}{\partial z^2} + \frac{\partial^2 \varepsilon_{zz}}{\partial y^2} = \frac{\partial^2 \gamma_{yz}}{\partial y \partial z} \]

\[ \frac{\partial^2 \varepsilon_{xx}}{\partial z^2} + \frac{\partial^2 \varepsilon_{zz}}{\partial x^2} = \frac{\partial^2 \gamma_{xz}}{\partial x \partial z} \]

\[ \frac{\partial}{\partial z} \left( \frac{\partial \gamma_{yz}}{\partial x} + \frac{\partial \gamma_{xz}}{\partial y} - \frac{\partial \gamma_{xy}}{\partial z} \right) = 2 \frac{\partial^2 \varepsilon_{zz}}{\partial x \partial y} \]

\[ \frac{\partial}{\partial y} \left( \frac{\partial \gamma_{xy}}{\partial z} + \frac{\partial \gamma_{yz}}{\partial x} - \frac{\partial \gamma_{xz}}{\partial y} \right) = 2 \frac{\partial^2 \varepsilon_{yy}}{\partial x \partial z} \]

\[ \frac{\partial}{\partial x} \left( \frac{\partial \gamma_{xy}}{\partial z} + \frac{\partial \gamma_{xz}}{\partial y} - \frac{\partial \gamma_{yz}}{\partial x} \right) = 2 \frac{\partial^2 \varepsilon_{xx}}{\partial y \partial z} \]
State the conditions under which the following is a possible system of strains:

\[
\begin{align*}
\varepsilon_{xx} &= a + b(x^2 + y^2) x^4 + y^4, & \gamma_{yz} &= 0 \\
\varepsilon_{yy} &= \alpha + \beta (x^2 + y^2) + x^4 + y^4, & \gamma_{zx} &= 0 \\
\gamma_{xy} &= A + B xy (x^2 + y^2 - c^2), & \varepsilon_{zz} &= 0
\end{align*}
\]
COMPATIBILITY CONDITIONS

State the conditions under which the following is a possible system of strains:

\[ \varepsilon_{xx} = a + b(x^2 + y^2) x^4 + y^4, \quad \gamma_{yz} = 0 \]
\[ \varepsilon_{yy} = \alpha + \beta (x^2 + y^2) + x^4 + y^4, \quad \gamma_{zx} = 0 \]
\[ \gamma_{xy} = A + B xy (x^2 + y^2 - c^2), \quad \varepsilon_{zz} = 0 \]

[Ans. \( B = 4; \ b + \beta + 2c^2 = 0 \)]